Resonators supporting Bessel beams

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A resonator is described which generates directly a Bessel Gauss beam as output. The resonator consists of an axicon mirror and a flat output coupler. Numerical simulation based on Fox-Li iteration in the cavity confirms the geometric-optic arguments to determine the resonance condition.

Introduction

It is known that an axicon transforms an incoming plane wave into a Bessel beam [1]. In Figure 1a a plane wave is incident from the left on the base plane AB of such an axicon. This axicon is assumed be thin. In the subsequent paragraphs the paraxial approximation is adopted. The plane wave is transformed into a conical wave, exhibiting an interference zone CFDE. In this zone the Bessel beam exists. If a mirror would be installed now in the plane CD, then the Bessel beam is reflected upon itself. This observation leads to the realization of the Bessel beam using a resonator.

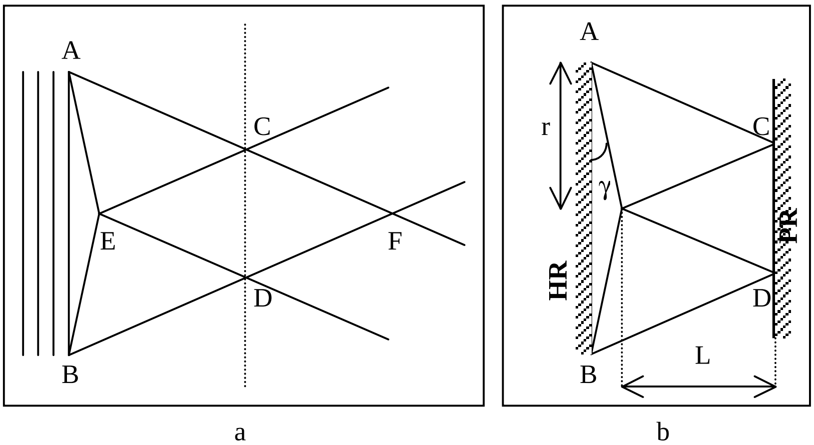

Figure 1: a) Transformation of a plane wave in a conical wave by transmission through an axicon.
b) Resonator generating a Bessel beam.

In Figure 1b the base plane AB of the axicon is made highly reflective, whereas the output coupler CD is partially reflective. The field is self-reproducing in two roundtrips. The resonance condition is

$$L = \frac{r}{2(n-1)\gamma} \qquad (1)$$

n being the refractive index of the axicon, γ its apex angle and $L = EF/2$. This expression is based on the formula expressing the angular deviation for a ray perpendicularly incident on a wedge with apex angle γ (see Figure 1a):

$\frac{AE}{EF} = \tan[(n-1)\gamma] \cong (n-1)\gamma$, see ref. 2. Also observe that AB = 2 x CD, i.e. that the interference interval of the conical wave on the plane mirror is half the size of the base of the axicon. The transverse dimension r of the axicon makes explicitly part of this self-consistency condition. In the usual case of resonators supporting Gaussian beams, the condition that rays reproduce themselves is independent of the mirror size, at least in the geometric optical limit.

In Figure 1, the transmissive axicon with the totally reflecting base can be replaced by a reflective axicon, leading to an alternative but equivalent realization of a Bessel resonator.

The fundamental Bessel wave is described by

$$E(x,y,z) = E_0 J_0(\alpha\rho) \quad \exp[-i(\omega t - \beta z)] \tag{2}$$

$$k = {}^{2\pi}\!/_{\lambda} = {}^{\omega}\!/_{c}$$

$$\rho^2 = x^2 + y^2$$

$$\alpha = \sqrt{k^2 - \beta^2} = k\sqrt{1 - \frac{\beta^2}{k^2}} \tag{3}$$

We now note that the triangle AEF in Figure 1a is similar to the triangle spanned by the wave vectors α, β and k leading to

$$\frac{\beta}{k} = \frac{2L}{\sqrt{4L^2 + r^2}}$$

giving the relation between the resonator parameters and the spatial frequency α of the Bessel function:

$$\alpha = \frac{kr}{\sqrt{4L^2 + r^2}} \tag{4}$$

If the resonance condition (1) is substituted in equation (4), the expression for the spatial frequency α becomes:

$$\alpha = \frac{(n-1)\gamma k}{\sqrt{1 + (n-1)^2 \gamma^2}} \tag{5}$$

which is now independent of L and r. This means that the spatial behaviour of the mode in the plane of the flat mirror is not dependent on the diameter of the axicon, only on its apex angle. This fact leads to a new class of equivalent resonators. All resonators having equal apex angles of their axicon mirror (and different resonators lenghts) have an identical shape of Bessel mode on their flat output coupler. For our particular set of experimental parameters eq (5) results in $\alpha = 7.2197\ \mathrm{mm}^{-1}$.

Numerical simulation

The reference plan to start the round trip in the resonator is chosen just before the output coupler plane CD of Figure 1b. Since it is the intention to validate the calculations on a CO_2 laser resonator, the following numerical values were fixed as follows:

λ = 10.6 micron
n = 2.4 (Zinc Selenide)
γ = 0.5 ° = 8.7 mrad
r = .75 “ / 2 = 9.53 mm

resulting in a cavity length L = 391.2 mm according to the resonance condition (1).
γ and r of the Zinc Selenide axicon correspond to a commercially available model, AR/AR coated for 10.6 micron.
The classical Fox-Li iteration algorithm was implemented to determine numerically the lowest loss mode of the resonator. The mode is sampled in the reference plane over a grid of 512x512 points. Upon transmission through the axicon, reflection at its rear side and subsequent second transmission through the axicon, a phase shift
exp [-2 i k (n – 1) γ r] is added to the wave. Typically 100 round-trips are required for the process to converge, starting from an arbitrary rotationally symmetric field distribution. The convergence is shown in Figure 2.

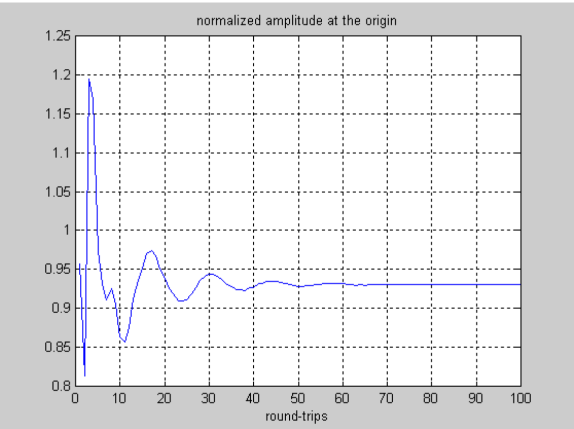

Figure 2: Normalized amplitude at the origin, as a function of the number of round-trips.

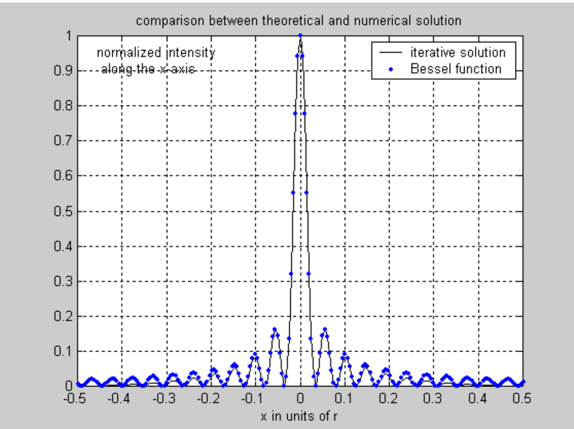

Figure 3: Comparison between theoretical and numerical solution. Normalized intensity along the x-axis. Full line: iterative solution; dots: Bessel function.

The intensity distribution is shown in Figure 3, where its shape is also compared to the ideal Bessel solution, equation (2) , with α given by equation (5). It can be observed that the real beam falls off faster than the Bessel beam, becoming virtually zero outside the interval [- r/2, r/2].
In Figure (4) the phase distribution along the x-axis is represented. If the beam would be an ideal Bessel beam, then the phase pattern would have been a block wave with phase jumps of π at the zeros of the Bessel function. Due to aperture diffraction however, phase distortion towards the edges of the plane mirror can be observed.
This is similar to the behaviour of the Bessel-Gauss resonator of ref. 3.

Finally, the (two-dimensional) Fourier Transform of the field at the flat mirror was calculated. Since the mode exhibits circular symmetry, so will also its Fourier Transform. In case of a pure Bessel mode, its transform is a Dirac delta function ring with radius α in the Fourier domain [4]. Fig. 5 shows the result for our numerical simulation. Due to aperture effects the radial part of the spatial frequency distribution is not an infinitely sharp spike. It is further evidence however that the resonator generates a quite good approximation of a Bessel mode, in fact it should be rather denominated a Bessel-Gauss mode, since if the Bessel function is numerically extracted from the intensity distribution by dividing by J_0 ($\alpha\rho$), a Gaussian- like function is effectively obtained.

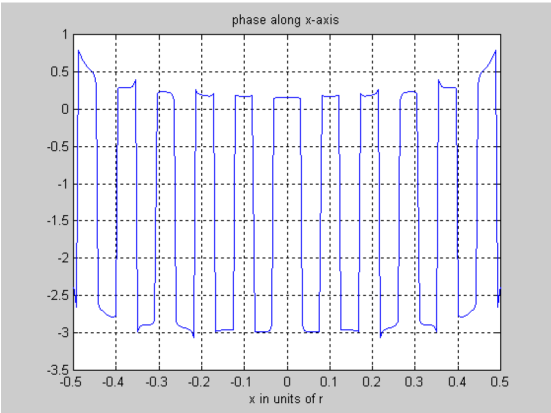

Figure 4: Phase along x-axis, where x is in units of r.

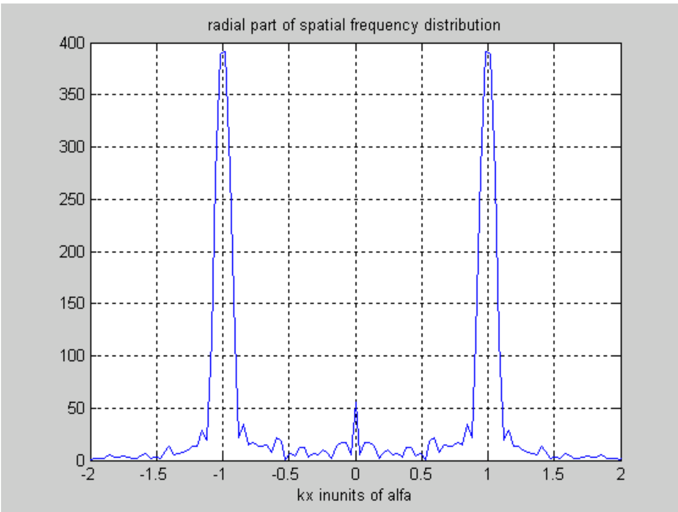

Figure 5: Radial part of spatial frequency distribution, where k_x is in units of α.

References

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