Application of birefringent filters in continuous-wave tunable lasers: a review

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In this review we summarize results of the use of birefringent filters for primary narrowing of laser emission spectra. We note some errors in publications on the problem. Simple practical recommendations on the choice and computation of parameters of the birefringent filters for a wide scope of problems are given.

INTRODUCTION

One of the most widely used methods of primary narrowing of the emission spectra of tunable cw lasers employs various modifications of the birefringent filter (BF) proposed by Lyot,[1] commonly referred to as the Lyot filters. Their successful uses are related to a number of their merits, namely, a narrow transmission passband (down to a few hundredths of nm), a fairly wide free spectral range (hundreds of nm), and no need of coatings (the filters can withstand powers up to hundreds of MW/cm^2). As far as nonselective losses are concerned, these filters appear to be the most perfect dispersive elements, their losses per single passage being not higher than 2–3%.

Birefringent filters have been treated in numerous papers and in a couple of reviews: a Chapter of a monograph *Tunable Lasers*[2] and a report submitted to the 3rd Los Alamos National Laboratory Conference on Optics.[3] However, the published papers contain some errors which we discuss below. In some cases optimal performance of these elements was not correctly understood either. That is why the application of this, as it seems, comprehensively studied device frequently required detailed comments on the computation procedure of its parameters, as well as on the analysis of its performance. Moreover, since the publication of the mentioned reviews a number of new results on the properties of laser BF have been obtained. These results are to be summarized and general practical recommendations are to be formulated for the choice and computation of the BF parameters for a wide scope of problems. These reasons motivated publication of the present review.

First we dwell upon common properties of a BF and upon its particular use inside laser cavities.

The simplest BF using interference of polarized light was proposed by R. W. Wood as far back as 1904. Its improved version is the Lyot filter.[1] The BF or, in other words, the interference-polarization filter adapted for laser applications was first proposed in Ref. 4.

A conventional Lyot filter is composed of a set of polarizers mutually parallel or crossed in pairs with birefringent plates inserted between them, each successive plate being twice as thick as the preceding one. The optic axes of the plates are parallel to each other and to the surfaces of the plates. The polarizer and the birefringent plate surfaces are oriented normal to the incident light beam. The optic axes make 45° with the polarizing directions of the polarizers, i.e., with the direction of the electric field vector of the transmitted light wave. The transmission spectrum of the Lyot filter for parallel orientation of the polarizers is given by

$$T=\frac{\sin^2\left(\frac{2^N\pi\Delta nl}{\lambda}\right)}{2^{2N}\sin^2\left(\frac{\pi\Delta nl}{\lambda}\right)}, \tag{1}$$

where l is the thickness of the thinnest plate, λ is the light wavelength in vacuum, $\Delta n=n_e-n_o$, n_o and n_e are the refractive indices for the ordinary and extraordinary rays, and N is the number of the birefringent plates.

The transmission function of the Lyot filter displays the main maxima and minima, their positions being determined respectively by the conditions

$$\frac{\Delta nl}{\lambda}=k,$$

where $k=1, 2, 3,\ldots,$ and

$$\frac{2^N\Delta nl}{\lambda}=k',$$

where $k'=1, 2, 3,\ldots,$ with $k'/2^N$ being a noninteger.

Inside the gap between a pair of neighboring main maxima, 2^N-1 minima are located along with weaker sidebands. The free spectral range $\Delta\nu_{FSR}$ of the Lyot filter is determined by the thickness of the thinnest plate

$$\Delta\nu_{FSR}=\frac{c}{\Delta nl},$$

where c is the velocity of light in vacuum. The bandwidth of the main maxima is controlled by the thickness L of the thickest plate: $\Delta\nu=c/2^{N-1}\Delta nL$.

The most important results of studying the BF in laser cavities are discussed in Refs. 5–15.

The design of the Lyot filter for intracavity applications was simplified owing to the threshold nature of laser action. It proved possible to reduce the number of its components and to eliminate the ideal polarizers. Filter surfaces at Brewster's angle with the incident light beams were used as polarizers. As a result, the conventional intracavity BF employed in tunable cw lasers consists of a few (most frequently, of three) plane-parallel plates with multiple thicknesses cut from a birefringent (uniaxial) crystal. The optic axes of the plates are set mutually parallel as before and may be either parallel to the plane of the plate

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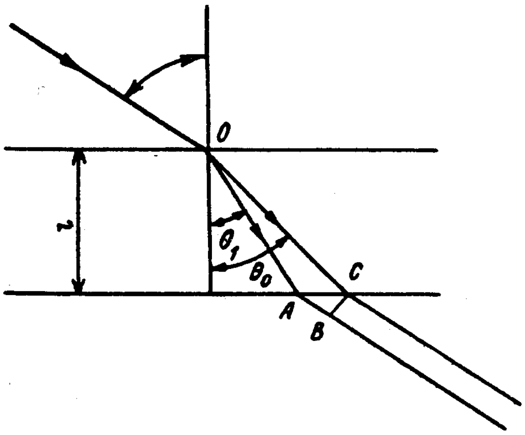

FIG. 1. Light rays in a birefringent plate: OC and OA are the ordinary and extraordinary rays, i is the angle of incidence, θ_1 and θ_0 are the refraction angles of the extraordinary and ordinary waves, and l is the plate thickness.

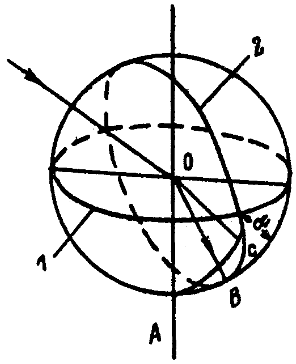

FIG. 2. Spherical geometry of the birefringent plate: OB is the wave vector in the medium, OC is the optic axis, OA is the normal to the plate surface, the plane of the figure is the plane of incidence, 1 is the plate surface, and 2 is the principal plane.

or make with it an angle β. The plates are tilted at Brewster's angle to the light beam. A BF of this type with $\beta=25°$ was first proposed in 1973 by associates of Coherent Radiation Inc. for use in dye lasers.[16] A patent, *Tuning Apparatus for an Optical Oscillator*, comprising a BF with $\beta=25°$ suitable for any laser, was claimed by the same authors in 1974.[17]

Now we discuss the essential properties of the BF using as an example a single-plate filter.

BIREFRINGENT PLATE OUTSIDE THE CAVITY

The operational properties of the filter are determined by the phase difference between ordinary (o) and extraordinary (e) waves after passing through a birefringent plate. Formation of the path difference Δ is illustrated by Fig. 1: $\Delta=OA\times n_e(\theta_1)+AB-OC\times n_o$. The phase difference (see, e.g., Ref. 18) is given by

$$2\delta=\frac{2\pi\Delta}{\lambda}=\frac{2\pi l}{\lambda}\left(\sqrt{n_e^2(\theta_1)-\sin^2 i}-\sqrt{n_o^2-\sin^2 i}\right), \quad (2)$$

where l is the thickness of the plate, i is the angle of incidence, θ_1 is the angle of refraction of the e wave, n_e and n_o are the refractive indices for the e and o waves respectively. For $\Delta n \ll n_o$ and n_e, Eq. (2) may be approximated by $2\delta \simeq 2\pi l\Delta n \sin^2\gamma/\lambda\cos\theta$, where θ is the mean value of θ_1 and θ_0 (θ_0 is the refraction angle of the o-wave), and γ is the angle between the optic axis and the wave vector of the e wave in the medium. For a light beam incident on the plate at Brewster's angle, which will be assumed henceforth, we can take $\cos\theta \approx \sin i$ and

$$2\delta \simeq \frac{2\pi l\Delta n \sin^2\gamma}{\lambda \sin i}. \quad (3)$$

The angle γ is commonly defined through other angles, $\mathscr{A}$ and β, more convenient for experimental measurements, where $\mathscr{A}$ is the angle between the plane of incidence and the plane containing the optic axis and the normal to the plate surface. To derive the required relations we use the spherical trigonometry formulas (see Fig. 2). In Fig. 2, the angle α between the plane of incidence and the principal plane of the phase plate is also indicated. As will be seen below, the magnitude of this angle is of considerable importance.

For the spherical triangle ABC we have: $\angle CAB=\mathscr{A}$, $\angle CBA=180-\alpha$, and for the arcs: $AC=90°-\beta$, $BC=\gamma$, $AB=\theta=90°-i$. We use hereafter the following relations which can be obtained by spherical trigonometry for ΔABC:

$$\cos\gamma=\sin\beta\cdot\sin i+\cos\beta\cdot\cos i\cdot\cos\mathscr{A}, \quad (4)$$

$$\tan\alpha=\frac{\sin\mathscr{A}}{\cos\mathscr{A}\cdot\sin i-\tan\beta\cdot\cos i}. \quad (5)$$

The polarization characteristics of coherent radiation can be readily described using the formalism of Jones matrices (see, e.g., Refs. 19 and 20). To recall their meaning and their derivation we deduce here the matrix for a bire-

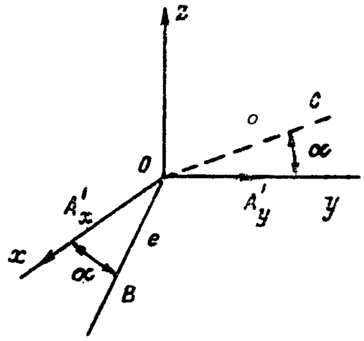

FIG. 3. Geometry of the birefringent plate: XOZ is the plane of incidence, OX is P polarization, OY is S polarization, ZOB is the principal plane, OZ is directed along the refracted beam, and α is the angle between the plane of incidence and the principal plane.

fringent Brewster's plate. It is useful also because this matrix often appears in publications in an erroneous form.

Consider a plate with an arbitrary direction of the optic axis. For simplicity, we take into account only the phase difference 2δ between the o and e waves rather than their absolute phases. Figure 3 explains the calculation of the matrix. The oscillation planes of e and o waves are arranged, respectively, in the principal plane of the phase plate (direction OB) and in the orthogonal direction (OC), both being perpendicular to the refracted ray.

After refraction at the Brewster surface, the amplitudes A'_x and A'_y of the P and S components become

$$A'_x=\tfrac{1}{n}A_x,\quad A'_y=\frac{2}{n^2+1}A_y.$$

where A_x and A_y are the respective input amplitudes.

We find next the amplitudes of the P and S components in the exiting beam. Since we do not care about absolute phase changes, we can assume that all the retardation is acquired by one of the waves. The o and e waves fall onto the exit surface of the plate with amplitudes

$$A_o=\left[\left(\frac{2}{n^2+1}\right)A_y\cos\alpha-\left(\frac{1}{n}\right)A_x\sin\alpha\right]e^{-2i\delta},$$

$$A_e=\left[\left(\frac{1}{n}\right)A_x\cos\alpha+\left(\frac{2}{n^2+1}\right)A_y\sin\alpha\right].$$

After refraction on the second Brewster surface the amplitudes of the $P(A''_x)$ and $S(A''_y)$ components become

$$\begin{aligned}A''_x=&\left[\left(\frac{1}{n}\right)A_x\sin\alpha-\left(\frac{2}{n^2+1}\right)A_y\cos\alpha\right]n\sin\alpha\cdot e^{-2i\delta}\\&+\left[\left(\frac{1}{n}\right)A_x\cos\alpha+\left(\frac{2}{n^2+1}\right)A_y\sin\alpha\right]n\cos\alpha\\&=A_x(\cos^2\alpha+\sin^2\alpha\cdot e^{-2i\delta})+qA_y\sin\alpha\\&\times\cos\alpha(1-e^{-2i\delta}),\end{aligned}$$

$$\begin{aligned}A''_y=&\left[\left(\frac{2}{n^2+1}\right)A_y\cos\alpha-\left(\frac{1}{n}\right)A_x\sin\alpha\right]e^{-2i\delta}\\&\times\cos\alpha\left(\frac{2n^2}{n^2+1}\right)+\left[\left(\frac{1}{n}\right)A_x\cos\alpha\right.\\&\left.+\left(\frac{2}{n^2+1}\right)A_y\sin\alpha\right]\sin\alpha\left(\frac{2n^2}{n^2+1}\right)\\&=qA_x\sin\alpha\cdot\cos\alpha(1-e^{-2i\delta})+q^2A_y(\sin^2\alpha\\&+\cos^2\alpha\cdot e^{-2i\delta}),\end{aligned}$$

where $q^2=(2n/n^2+1)^2$ is the power transmittance for the S polarized wave. The matrix for a birefringent Brewster plate

$$\mathscr{M}=\begin{pmatrix}A & B\\ C & D\end{pmatrix}$$

can be derived from

$$\begin{pmatrix}A & B\\ C & D\end{pmatrix}\times\begin{pmatrix}A_x\\ A_y\end{pmatrix}=\begin{pmatrix}A''_x\\ A''_y\end{pmatrix}.$$

It has the form

$$\mathscr{M}=\begin{pmatrix}\cos^2\alpha+\sin^2\alpha\cdot e^{-2i\delta} & q\sin\alpha\cdot\cos\alpha(1-e^{-2i\delta})\\ q\sin\alpha\cdot\cos\alpha(1-e^{-2i\delta}) & q^2(\sin^2\alpha+\cos^2\alpha\cdot e^{-2i\delta})\end{pmatrix}. \qquad (6)$$

As follows from this result, the filter properties are mostly determined by the phase difference acquired by the two waves in the plate and also by the angle α between the plane of incidence and the principal plane of the birefringent plate.

Note that the same matrix was employed in Refs. 7, 13, and 14, whereas in Ref. 5 a similar matrix was used with the angle α being replaced by an angle ψ between the plane of S polarization and the projection of the electric field of the o wave on the plane normal to the incident beam. Estimates for the crystalline quartz show the difference between angles ψ and α to be rather small ($<2°$) but still of significant importance. Since Ref. 5 was the first paper devoted to characterization of the BF with Brewster's surfaces in a tunable laser, the mentioned inaccurate determination of the angle α resulted in use of the incorrect angle for the Jones matrices for the BF plates and for computing the phase difference in a number of subsequent papers.[6,9,10] We discuss the consequences of this substitution below.

Note also that the authors of Ref. 9 when deriving the matrix for the birefringent plate did not use the idea of the phase difference between the e and o waves. As a result, they ignored a part of the pathlength of one of the rays (AB in Fig. 1), whereas it is just the phase difference that determines the result of interference of two light waves.

BIREFRINGENT BREWSTER'S PLATE INTRACAVITY

Many properties of an intracavity BF can be understood by the example of a single Brewster phase plate. The problem is usually solved assuming entire restoration of the polarization state of the light after the round-trip over the laser cavity. The eigenvectors and eigenvalues of the matrix equation

$$\mathscr{M}\cdot\mathbf{x}=\Lambda\cdot\mathbf{x}, \tag{7}$$

are to be found. Here

$$\mathscr{M}\equiv\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{vmatrix}$$

is the filter matrix,

$$\mathbf{x}=\begin{vmatrix} X_1 \\ X_2\end{vmatrix}$$

is the eigenvector, X_1 and X_2 are its P and S components, and Λ is the eigenvalue (the transmittance of the filter is $|\Lambda|^2$). A nonzero solution of the equation exists when the matrix determinant $|\mathscr{M}-\Lambda E|$ (E is the unit matrix) equals zero. This condition yields the equation for the eigenvalues which may be complex:

$$\Lambda^2-\Lambda(m_{11}+m_{22})+m_{11}m_{22}-m_{12}m_{21}=0, \tag{8}$$

$$\Lambda_{1,2}=\frac{m_{11}+m_{22}\pm\sqrt{(m_{11}-m_{22})^2+4m_{12}m_{21}}}{2}. \tag{9}$$

For a birefringent Brewster plate the matrix $\mathscr{M}$ is given by Eq. (6) and $\Lambda_{1,2}$ have the form

$$\Lambda_{1,2}=\frac{e^{-i\delta}}{2}\,[\cos\delta(1+q^2)+i\sin\delta(1-q^2)\cos 2\alpha$$
$$\pm\sqrt{\cos^2\delta(1+q^2)^2-\sin^2\delta(1-q^2)^2\cos^2 2\alpha-4q^2+i\sin 2\delta(1-q^4)\cos 2\alpha}. \tag{10}$$

The eigenvector components can be found from the equations

$$(m_{11}-\Lambda)X_1+m_{12}X_2=0,$$
$$m_{21}X_1+(m_{22}-\Lambda)X_2=0. \tag{11}$$

As the simplest example, consider a ring-laser cavity with the plate at $\alpha=45°$. For this the matrix of the birefringent Brewster plate is

$$\mathscr{M}_{45°}=e^{-i\delta}\begin{pmatrix}\cos\delta & iq\sin\delta \\ iq\sin\delta & q^2\cos\delta\end{pmatrix}, \tag{12}$$

and the eigenvalues are

$$\Lambda_{1,2}=e^{-i\delta}\,\frac{(1+q^2)^2\cos\delta\pm\sqrt{(1+q^2)^2\cos^2\delta-4q^2}}{2}. \tag{13}$$

The equations for the eigenvector components are

$$(e^{-i\delta}\cos\delta-\Lambda)X_1+e^{-i\delta}iq\sin\delta\cdot X_2=0,$$
$$e^{-i\delta}iq\sin\delta\cdot X_1+(e^{-i\delta}q^2\cos\delta-\Lambda)X_2=0. \tag{14}$$

For $\delta=\pi k$, we have $\Lambda_1=1$ and $\Lambda_2=q^2$. Now the polarization states related to these eigenvalues, i.e., the eigenvector components X_1 and X_2, can be easily found:

$$(1-\Lambda)X_1=0,\quad (q^2-\Lambda)X_2=0.$$

Thus for $\Lambda=1$ ($X_2=0$ and $X_1\neq0$), we have P polarization, while for $\Lambda=q^2$ ($X_1=0$ and $X_2\neq0$) we have S polarization. Therefore at $\delta=\pi k$ the P-polarized wave exhibits minimum losses, which corresponds to the main transmission maxima of the phase plate.

The wavelength of the light characterized by the smallest losses at this value of δ is given by Eq. (3) and equals

$$\lambda_k=\frac{\Delta n l\sin^2\gamma}{k\sin i}, \tag{15}$$

where the integer k is the order of the main transmission maximum.

For δ close to πk we have

$$\Lambda_1\simeq e^{-i\delta}\left(1-\frac{\delta^2}{2}\cdot\frac{1+q^2}{1-q^2}\right),$$
$$\Lambda_2\simeq e^{-i\delta}\left(1+\frac{\delta^2}{2}\cdot\frac{1+q^2}{1-q^2}\right)q^2, \tag{16}$$

and the light polarization becomes elliptic. As δ deviates from πk, $|\Lambda_1|^2$ decreases while $|\Lambda_2|^2$ grows and at certain $\delta=\delta_*$ the losses for both waves appear to be the same. Circular polarization corresponds to the eigenvalue $\Lambda_*=|\Lambda_1|=|\Lambda_2|=q$ and $T_{\min}=|\Lambda_*|^2=q^2$ is the minimum filter transmission. The value

$$2\delta=2\delta_*=2\arccos\left(\frac{2q}{1+q^2}\right) \tag{17}$$

is conventionally taken for the transmission bandwidth of the birefringent Brewster plate in a ring cavity. When q is close to unity, $\delta_*\simeq1-q$. It is worth noting that the worse the polarizing properties of the surface, the narrower the transmission passband and the higher the minimum transmission. The dependences of the filter transmittance $T=|\Lambda|^2$ on δ for different values of q are shown in Fig. 4. Note that the discontinuities of the transmission curves in Fig. 4 are observed only at $\alpha=45°$.

The corresponding transmission bandwidth of the filter is

$$\Omega\delta\lambda=\frac{2\lambda^2\delta_*\sin i}{\pi l(n_e-n_o)\sin^2\gamma}. \tag{18}$$

The free spectral range of this plate is given by

$$\Delta\lambda=\frac{\lambda^2\sin i}{l(n_e-n_o)\sin^2\gamma}. \tag{19}$$

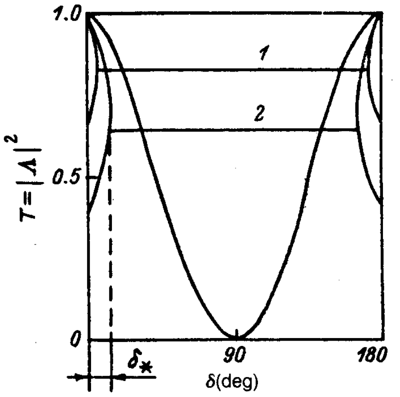

FIG. 4. Transmittance T of the filter vs the phase difference δ for different q (according to Ref. 6): $q^2=0.81$ (*1*), 0.64 (*2*).

Operation of a BF in a Fabry–Perot cavity differs from that in a ring resonator since in the former the light passes through the plate twice per round-trip and the matrix $\mathscr{M}$ is a product of the single-plate matrix by itself. Now the minimum transmittance $T_{\min}$ becomes q^4 rather than q^2 and the transmission spectrum of the Brewster phase plate acquires a sideband located between the main transmission peaks. This maximum is related to the light wave undergoing losses only on two of the four Brewster surfaces per round-trip.

Consider again the simplest case of a ring cavity. So far we assumed $\alpha=45°$. Now we examine the effect of deviation of α from this value which occurs under tuning when the angle $\mathscr{A}$ is varied. As can be seen from the matrix $\mathscr{M}$, Eq. (6), the transmission of the P and S components by the birefringent Brewster plate at $e^{-2i\delta}=+1$ does not depend on α and equals, respectively, $\Lambda_1^2=1$ and $\Lambda_2^2=q^2$. The selectivity of this plate increases with narrowing of the central transmission peak and with decrease of the transmittance near the transmission minimum. The minimum transmission occurs in particular at $e^{-2i\delta}=-1$. Let us discuss this case.

Recall that at $\alpha=45°$ when the latter condition is satisfied, $\Lambda_1^2=\Lambda_2^2=q^2$. The matrix of the birefringent Brewster plate, $\mathscr{M}_{\min}$, at $e^{-2i\delta}=-1$ has the form

$$\mathscr{M}_{\min}=\begin{pmatrix}\cos^2\alpha-\sin^2\alpha & 2q\sin\alpha\cos\alpha\\ 2q\sin\alpha\cos\alpha & q^2(\sin^2\alpha-\cos^2\alpha)\end{pmatrix}. \quad (20)$$

Let $\alpha=45°+\vartheta$. Then we have

$$\Lambda_{1,2}^{\min}=\frac{-(1-q^2)\sin 2\vartheta\pm\sqrt{(1-q^2)^2\sin^2 2\vartheta+4q^2}}{2}. \quad (21)$$

For small ϑ, $\Lambda_{1,2}^{\min}\simeq(q^2-1)\vartheta\pm q$ and

$$|\Lambda_1|^2_{\min}\approx q^2+2\vartheta(q^2-1)q,$$

$$|\Lambda_2|^2_{\min}\approx q^2-2\vartheta(q^2-1)q. \quad (22)$$

TABLE I.

β	$\mathscr{A}$
0	40
15	33
30	26
45	15
67.4	50

Therefore, one of the transmission eigenvalues of the plate near the minimum transmission for α close, but not equal, to 45° becomes higher than the transmission at $\alpha=45°$ and the filter selectivity decreases.

It is evident that for $\alpha=0$ the transmittance of the plate is characterized by two eigenvalues which are independent of δ: $|\Lambda_1^2|=1,q^4$, i.e., the plate displays no selectivity at all.

Finally, we can examine variations of the rate of transmission decrease with the growth of δ near the maximum at different values of α and make sure that this dependence appears to be the steepest at $\alpha=45°$. Therefore, both the contrast of the transmission function and the bandwidth of the main maximum is optimal at this value of α, and the BF as a selector becomes most efficient. Below we show that at $\alpha=45°$ the sidebands of the transmission function, which arise when proceeding from a single plate to a pile, become weaker as well. Note that the dependence on α of the transmission function contrast, of the bandwidth of the transmission maxima, and of the sideband intensities do not alter with orientation of the optic axis of the phase plates. The dependence of the transmission in the strongest sideband and of the background transmission on the angle of rotation was reported in Ref. 11.

The correct inference on the optimal angle value, $\alpha=45°$, has been drawn in several papers.[7,11–15] At the same time, some articles were not entirely correct in this respect. For example, in Ref. 5 it was stated that the optimal angle of the BF is $\psi=45°$ which, as pointed out above, is close to but still differs from α by definition. In Refs. 6, 9, 10, and 21 the conclusion was drawn that the filter is most efficient at $\mathscr{A}=45°$. This angle may considerably differ from α. All these considerations are useful if calculation of a filter becomes necessary. In Table I we give, for convenience, the $\mathscr{A}$ values corresponding to $\alpha=45°$ for different β [see Eq. (6)] for a quartz Brewster plate ($n\simeq1.54$). In practice, the angle $\mathscr{A}$ is commonly used rather than α; the filter is calculated for the optimal angle $\alpha=45°$ and then the appropriate angle $\mathscr{A}$ is determined.

For a tunable laser, the tuning off from the wavelength, assumed to be the optimum, inevitably implies a drift from $\alpha=45°$. The rate of this drift ($d\alpha/d\lambda$) is mainly determined by the orientation of the optic axis inside the plate, i.e., by angle β. For a fairly long time (for more than a decade after the appearance of Ref. 5) only the BFs with either $\beta=25°$ (Refs. 5,6,10) or $\beta=0°$ (Refs. 7,9) were used. The choice of $\beta=25°$ can be explained by a smaller rate of drift from the optimal BF orientation while the choice of $\beta=0°$ is governed by convenience of the filter fabrication. However, the filters prove to operate satisfac-

TABLE II.

β	$(d\lambda/d\alpha)$
0	3.3
10	4.8
20	6.4
30	8.2
40	10.6
50	14.2

torily in both cases, providing most of the available tunability range and an acceptable bandwidth of the laser output spectrum.

The problems of choice of the angle β were discussed in Refs. 11–13, 15, and 21. Consider some of the proposed versions.

In Ref. 11 the effect of the angle β on the tunability coefficient, $d\lambda/d\mathscr{A}$, on the free spectral range of the BF, and on the coefficient $d\lambda/d\beta$ was studied. It was shown that the tunability coefficient $d\lambda/d\mathscr{A}$ varied from 3.3 to 25.5 nm/deg with an increase of the angle β from 0 to 53.5°. It was pointed out that the increase of the angle β under fixed thickness of the first plate can alter the free spectral range of the BF by several fold. The dependence of $d\lambda/d\alpha$ on β obtained in Ref. 12 is presented in Table II. The choice of the angle β was proposed in Ref. 12 on the basis of admissible contrast decrease in the BF transmission function, T_{max}/T_{min}, within the tuning range. For example, accepting the twofold admissible decrease in the contrast, variations of the angle α should not exceed $\pm 14°$ around α_{opt}. In spite of the approximate character of this criterion it should be noted that the filter designed under the assumption of a twofold admissible decrease in the BF transmission function contrast does not undergo jumps of laser emission from the main transmission maximum to a sideband in a wide range of gains and bandshapes of tunable cw lasers. Thus, if the filter is required for the 90-nm tuning range, we have $\Delta\lambda/\Delta\alpha=90/(2\times 14)\simeq 3.2$ nm/deg, and according to Table II we may choose $\beta=0°$ (Table II), i.e., the optic axis may be parallel to the phase plate surface. If tuning within the 150-nm range is required, we have $\Delta\lambda/\Delta\alpha=150/(2\times 14)\simeq 5.6$ nm/deg, and the filter with $\beta=15°$ will now be most suitable, etc. In the method described in Ref. 11 and later in Ref. 12 the angle β was proposed to be in conformity with the tunability range of the BF.

In Ref. 21 the detuning rate of the BF near the orientation corresponding to $\mathscr{A}=45°$ was computed as a function of β. It was concluded that the detuning rate, $d\lambda/d\mathscr{A}$, is highest at $\beta=30$–$35°$, and this range of angles was regarded as optimal. This result appeared to be erroneous since, first, the computations were carried out for $\mathscr{A}=45°$ rather than for $\alpha=45°$ and, second, the detuning rate for each value of β should be computed near the appropriate value of $\mathscr{A}_{opt}(\beta)$ (see Table I). As follows from the numerical results of Ref. 11 the derivative $d\lambda/d\mathscr{A}$ displays a maximum within the range $50°<\beta<60°$.

The choice proposed in Ref. 15 was $\beta=18°50'$. This quantity was derived from the condition of simultaneously meeting two equalities, $\alpha=45°$ and $\gamma=45°$, in the center of the BF tunability range. While the equality $\alpha=45°$ requires no additional comments, the problem of choice of γ is not so obvious. According to Ref. 15, the derivative

$$\frac{d\lambda}{d\gamma}=\frac{(n_e-n_o)d\sin 2\gamma}{m\sin\gamma},$$

should display maximum in the center of the tuning range, whence $\gamma=45°$. The proposed filter with $\beta=18°50'$ was compared within a tuning range of 100 nm with the filter with $\beta=0°$ and it was concluded that the proposed filter was superior since it exhibited $\Delta\alpha\simeq 15°$, whereas for the second filter $\Delta\alpha\simeq 28°$. It should be noted, however, that on the basis of the criterion of the least variation of α in a given tuning range, the filters with $\beta>18°50'$ appear preferable (provided the requirement of $\alpha=45°$ in the center of the tuning range is retained). Moreover, the least variation of α in a given tuning range is displayed by the filter with the angle β specified by $d\alpha/d\mathscr{A}=0$.

Since

$$\frac{d\alpha}{d\mathscr{A}}=\frac{\sin^2\alpha}{\sin^2\mathscr{A}}(\sin i-\tan\beta\cos i\cos\mathscr{A}), \tag{23}$$

it can be shown, taking into account Eq. (5), that $d\alpha/d\mathscr{A}=0$ at $\alpha=45°$ is satisfied provided that

$$\cos\beta=\frac{1}{\sqrt{2(1+\tan^2 i)}}=\frac{1}{\sqrt{2(1+n^2)}}, \tag{24}$$

whence for the quartz Brewster plate we obtain $\beta=67.4°$. It should not be concluded, however, that the angle β should be chosen only for these reasons. To the forefront can be brought other requirements, in particular, those related to the detuning rate or to a certain selectivity under maximum accessible thickness of the thickest BF plate.

This completes the section on the single birefringent Brewster plate inside the laser cavity. The essential properties of the filters based on this type of plate are revealed in the properties of a single plate. However, a number of new features appear if a stack of plates is used.

SYSTEM OF INTRACAVITY BIREFRINGENT BREWSTER PLATES

As a spectral selector it is more common to use an assembly of birefringent Brewster plates (usually three) of different thicknesses with mutually parallel optic axes. Proceeding to the assembly of plates is brought about by the need to make the passband of the main maxima narrower and to reduce the transmission in the region of the transmission minima. The plates with multiple thicknesses are traditionally employed in these sets, although, as shown in Ref. 12, this is not obligatory.

As follows from Eq. (2), the thicknesses of plates having their transmission maximum at a certain wavelength $\lambda=\lambda_k$ should satisfy the equalities: $l_1=m_1\lambda_k S$, $l_2=m_2\lambda_k S,\ldots,l_N=m_N\lambda_k S$, where N is the number of plates, m_i is an integer, and

$$S=\frac{1}{\sqrt{n_e^2(\theta_1)-\sin^2 i}-\sqrt{n_o^2-\sin^2 i}}.$$

From here we can obtain relations that must be satisfied: $l_2=m_2l_1/m_1$, $l_3=m_3l_1/m_1,...,l_N=m_Nl_1/m_1$. It is evident that m_i/m_1 is not necessarily an integer. If the thicknesses of the plates are integral multiples, the free spectral range is determined by the thickness of the thinnest plate. Otherwise the free spectral range is increased by a factor of r where r is the least integer for which the values of rm_i/m_1 are integers for all N. It is interesting to note that a filter whose order values m_i are prime numbers has only one main transmission maximum. Thus, the free spectral range of the filter with a noninteger ratio of the plate thicknesses is determined by the particular values of their thicknesses.

For simplicity we assume that all plates have the same mean refractive index (the same q), the axes of the plates are mutually parallel, and the arrangement of the plates in the cavity corresponds to $\alpha=45°$. Analyzing the operation of the filter in the cavity we also take into account the Brewster surfaces of isotropic media; such interfaces are created, for example, by the active-medium jet in liquid-state lasers. The optical system is assumed to contain no ideal polarizers. Let N_e be the total number of filter elements including the isotropic Brewster plates. For simplicity we treat the filter with an integral multiple of the plate thicknesses and assume that the position of the mth-order transmission maximum of the thinnest plate of thickness l_1 coincides with the mr_ith-order maximum of the plate of thickness r_il_1. In this treatment we mainly follow the results of Ref. 7.

When deriving the full matrix of the system the succession of passing the filter elements should be taken into account. If the matrices of the single elements are $\mathscr{M}(r_1\delta),\mathscr{M}(r_2\delta),...,\mathscr{M}(r_{N_e}\delta)$, and the light passes them successively as they are written, the matrix of the composite filter, $\mathscr{M}_c$, in a ring cavity will be the product of these matrices with the inverse order of multipliers. The matrix of the same filter in a Fabry–Perot cavity will have the form

$$\mathscr{M}_c=\mathscr{M}(r_1\delta)\cdot\mathscr{M}(r_2\delta)\cdot...\cdot\mathscr{M}(r_{N_e}\delta)\cdot\mathscr{M}(r_{N_e}\delta)\cdot...$$
$$\cdot\mathscr{M}(r_2\delta)\cdot\mathscr{M}(r_1\delta).$$

Ignoring the common phase factor, the matrix $\mathscr{M}(r_i\delta)$ can be written as follows

$$M(r_i\delta)=\begin{pmatrix}\cos(r_i\delta) & iq\sin(r_i\delta)\\ iq\sin(r_i\delta) & q^2\cos(r_i\delta)\end{pmatrix}, \quad (25)$$

where δ is determined by the thickness of the thinnest plate $(r_1=1)$. It can be easily shown[7] that the matrix of the composite system at $\alpha=45°$ can be presented in the form

$$\mathscr{M}_c=\begin{pmatrix}a & ib\\ ic & d\end{pmatrix}, \quad (26)$$

where a, b, c, and d are some real coefficients. The equation for eigenvalues which governs the transmission of the composite filter in the cavity has, as before, the form: $\mathscr{M}_c\mathbf{x}=\Lambda\mathbf{x}$. The eigenvalues can be found from $\Lambda^2-\Lambda(a+d)+bc+ad=0$:

$$\Lambda_{1,2}=\frac{a+d\pm\sqrt{(a+d)^2-4(bc+ad)}}{2}. \quad (27)$$

For $\delta=nk$, for the ring cavity we have

$$\mathscr{M}_c=\begin{pmatrix}1 & 0\\ 0 & q^{2N_e}\end{pmatrix}$$

so that

$$\Lambda_{1,2}=\frac{1+q^{2N_e}\pm(1-q^{2N_e})}{2},$$

$$|\Lambda_1|^2=1, \quad |\Lambda_2|^2=q^{4N_e}. \quad (28)$$

The transmission of S-polarized light at the main maxima of a composite filter is significantly lower than for a single-plate filter (q is obviously less than unity). For $|a+d|<2q^{N_e}$, the eigenvalues become complex since the radicand in Eq. (27) is negative. In this case the two waves degenerate into a single wave with the transmission being equal to

$$T=\tfrac{1}{4}[(a+d)^2+4q^{2N_e}-(a+d)^2]=q^{2N_e}, \quad (29)$$

whence it follows that the minimum light transmission for a composite filter is also lower than for a single-plate one. It is noteworthy that the action of an isotropic plate is similar in this sense to that of the birefringent plate.

To estimate the bandwidth of the main maximum, we have to find the value of $\delta-\delta_*$ for which $|a+d|=2q^{N_e}$. For this, in turn, we need to know the dependence of $\mathrm{Tr}\mathscr{M}_c=(a+d)$ on δ. It can be shown[7] that

$$\mathrm{Tr}\,\mathscr{M}_c=a+d$$
$$=(1+q^{2N_e})\cos[(r_1+r_2+...+r_N)\delta]+K, \quad (30)$$

where N is the number of the birefringent plates in the filter, and K is the sum of terms having the common factor $(1-q^2)^2$ and containing a product of at least two sines, $\sin(r_i\delta)\sin(r_k\delta)$. For $(1-q^2)\ll 1$, term K may be neglected, so that

$$a+d\cong(1+q^{2N_e})\cos\sum_{i=1}^{N}r_i\delta. \quad (31)$$

This approximation is valid when the argument of cos is small or close to $k\pi$. It should be stressed that N_e in Eq. (31) is the total number of elements including the isotropic plates, whereas the summation is made only with respect to N, i.e., to the number of the birefringent plates. The approximation used in Eq. (31) usually is justified since as a rule q^2 is close to unity (e.g., for quartz $q^2=0.83$). Equation (30) is presented here for estimating the width of the principal maxima.

Thus,

$$a+d=2q^{N_e}\approx(1+q^{2N_e})\cos\sum_{i=1}^{N}r_i\delta_*,$$

whence

$$\sum_{i=1}^{N} r_i \delta_* \approx N_e(1-q)$$

and

$$2\delta_* \approx \frac{2N_e(1-q)}{\Sigma_{i=1}^{N} r_i}. \qquad (32)$$

The bandwidth of the main transmission maximum is given by

$$2\delta\lambda \approx \frac{2\lambda^2 N_e(1-q)\sin i}{\pi l(n_e-n_o)\sin^2\gamma \cdot \Sigma_{i=1}^{N} r_i}. \qquad (33)$$

For a single plate, as shown above, we have $\delta_* \simeq (1-q)$. As is evident from Eq. (32), introduction of isotropic plates broadens the main maxima although the level of T where this width is measured becomes lower.

An additional feature of the cavity with the assembly of plates is the appearance of the transmission sidebands at wavelengths where the value $\mathrm{Tr}\mathscr{M}_c = a + d \approx (1 + q^{2N_e})\cos\Sigma_{i=1}^{N} r_i\delta$ reaches its maximum, i.e., when $\delta\Sigma_{i=1}^{N} r_i = 180° \, m$, where m is an integer. This condition implies that the sidebands arise when the phase difference produced by an effective plate of the same net thickness is a multiple of 2π. These maxima are related to the incomplete polarization decoupling of the filter components. In the Fabry–Perot cavity the sidebands are half as far apart as in the ring cavity (N is doubled). The numerical treatment[11] shows the intensity of the sidebands to be the least at $\alpha=45°$, rather than at $\mathscr{A}=45°$ as claimed in Ref. 6. In general, the intensity of each sideband is determined by the ratio of thicknesses of the BF plates. Let us dwell upon the choice of the thickness ratio (TR) in more detail.

The filter proposed in Ref. 16 has a TR of 1:4:16. Filters with this TR value are used now most frequently,[5,10,17] in particular, in commercial cw dye lasers.[22] Another common combination of the thicknesses, 1:2:15,[6,9] is also employed in commercial cw dye lasers.

It was concluded in Ref. 6 that, from the viewpoint of the height of the most intense sideband, the optimum TR is 1:2:9. According to Ref. 6, the transmission of such a filter in the mentioned sideband is ~60%, whereas the corresponding transmission of the filter with a TR of 1:2:15 is ~80%. These figures do not completely agree with the results of Ref. 11, where the transmission in the strongest sideband was calculated for filters with a TR of $1{:}r{:}l$ where $r=2$–4 and $l=9$–16. It has been shown that the weakest sidebands are displayed by filters with a TR of 1:3:10 and 1:3:11 (their transmittance in the strongest sideband is 65%, whereas for the 1:2:9 filter it reaches 67%). The transmittance of traditional filters in the most intense sidebands reaches 78% (1:4:16) and 82% (1:2:15).

Thus, lower transmission in the strongest sideband is displayed by filters with a thinner third plate (the first plate being of the same thickness) and hence with lower spectral selectivity. The filters with higher selectivity (1:4:16, 1:2:15) have higher transmission in the strongest sideband. According to Ref. 11, as an optimal version of the integral-multiple TR a filter may be proposed with the

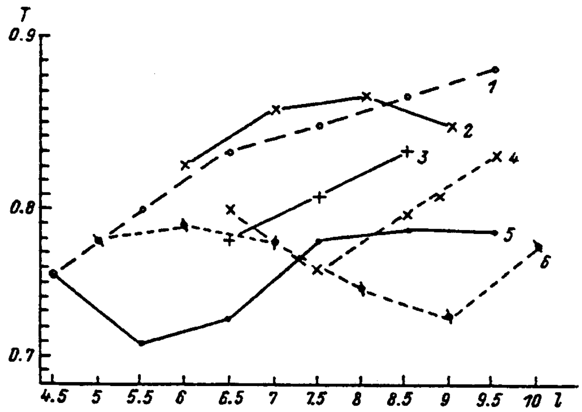

FIG. 5. Transmittance in the strongest sideband for different BFs with noninteger ratio of the plate thicknesses: 1—1:2:l, 2—1:2.5:l, 3—1:3:l, 4—1:2.5:l, 5—1:1.5:l, and 6—1:1.5:l.

thickness of the third plate differing from that of the first one by a factor intermediate between the maximum (15,16) and the minimum (9,10,11) values. Such 1:4:13 filter displays a 69% transmittance in the strongest sideband, close to that in the weakest one.

The use of noninteger TR for improving the laser BF was first discussed in Ref. 12 where, in particular, the transmission function was reported for a filter with a TR of 1:1.5:5.5 and with a first-plate thickness of 930 μm. The spacings between the main transmission maxima of this filter were shown to be equal to corresponding spacings of the filter with a first-plate thickness of 465 μm and with an integer TR. The strongest-transmission sideband of the 1:1.5:5.5 filter was 71%. Figure 5 presents the results of calculation of the strongest sideband of the $1{:}k{:}n$ filters ($k=1.5$, 2, 2.5, and 3; $n=4.5$–10) using the method proposed in Ref. 11. As follows from Fig. 5, the weakest transmission sidebands are displayed by the filters 1:1.5:5.5 (71%), 1:1.5:6.5 (72.5%), 1:1.5:9 (73%), and 1:1.5:8 (74%).

A four-component filter with a noninteger TR was proposed in Ref. 23 where a comparative analysis of the 1:1.5:3:9 and 1:2:6:18 filters intended for use in a Ti:sapphire laser was carried out. It has been shown that these filters display identical free spectral ranges but the first plate of the filter with noninteger TR is twice as thick (0.5 instead of 0.25), simplifying its fabrication. According to Ref. 23, the transmittance in the strongest sidebands is almost the same for both filters and reaches $\simeq$80%.

In Ref. 24 the thicknesses of the filter plates were proposed in accordance with the proportion

$$pl{:}(p+1)l{:}(p+2k)l{:}(p+3+k)l, \qquad (34)$$

where $p>1$ is an integer, k is also an integer,

$$l = \frac{\lambda^2 \sin i}{\Delta\lambda(n_e-n_o)\sin^2\gamma},$$

$\Delta\lambda$ is the free spectral range of the filter, and λ is the wavelength in the center of the tuning range. This filter uses only three plates, the third one being chosen in accor-

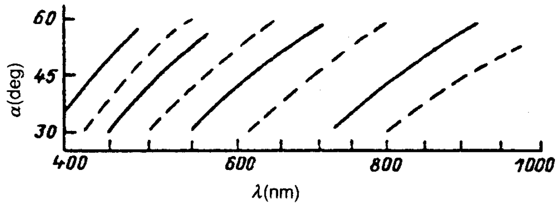

FIG. 6. Computed tuning curves for the set of two birefringent filters designed for operation in tunable cw lasers.[27]

dance with either of the two last terms in Eq. (34). It was claimed that, compared with the filter previously used, the proposed TR permits one to considerably increase the thickness of the thinnest plate and thus to significantly reduce the cost of the filter fabrication.

Thus, for the three-component filters with noninteger TR the optimum ratio of thicknesses of the first two plates is 2:3. This allows one to retain the free spectral range of the filter by doubling the thickness of the first plate. The choice of the third-plate thickness is of no great importance here since the intensity variation of the largest sideband is small for different combinations.

Concluding the treatment of the laser BF performance we discuss next the papers where some important aspects of the BF applications are considered.

A simple and reliable method of adjustment of a multicomponent filter was proposed in Ref. 25. The parallelism of optic axes of the plates is verified by measuring the tuning rate with only one plate, then with two, with three, etc. The filter rotation mechanism usually includes a calibrated micrometer screw. The tuning rate is measured in relative units using the micrometer screw and the wavelength meter (spectrometer or monochromator). After inserting each successive plate into the holder, one should fix the same wavelength and by rotating this plate achieve the same tuning rate as with the plates placed before. Besides, the filter must provide smooth tuning of the laser emission wavelength in a specified spectral range with one, with two, and with three plates.

However, if the assigned tolerances for the plates or holders are not adhered to (the values of these tolerances are given in Ref. 10), it may happen that the same tuning rates of the filters with different number of plates cannot be obtained. In this case the filter mounting should provide minimal difference in the tuning rates for a different number of plates.

The polarization eigenstates in a cavity with a partial polarizer and a phase plate were studied in Ref. 26. It has been shown that the polarization eigenstates in such a laser cavity reach the steady state after from a few tens up to a few hundreds of passages when q changes from 0.1 to 0.9. Therefore, the described properties of the BF are retained in pulsed tunable lasers with a large effective number of passages (of the order of 100).

A set of two BFs capable of tuning the cw laser emission at least in the range of 400–1000 nm was reported in Ref. 27. In Fig. 6 the computed tuning curves of the set are presented (solid line—for $l_1=400$ μm and broken line—for $l_1=450$ μm; for both filters $\beta=15°$). As is evident from Fig. 6, within any spectral region inside the 400–1000-nm range the wavelength may be tuned in turn with one or both BFs by rotating each of them near the position where the transmission function exhibits maximum contrast.

In Refs. 28–31 the BFs were used in gas lasers for tuning over the lasing lines. The only essential condition for the successful operation of the filter here is noncoincidence of the free spectral range with the spacing between the lasing lines. Note that for a relatively small free spectral range the tuning to any lasing line can be implemented near the optimum filter orientation ($\alpha=45°$). In this case the different main transmission maxima will correspond to different lasing lines.

Finally, we discuss the BF intended for controlling the emission spectrum of wideband tunable lasers, in particular, the synchronously pumped dye or color-center lasers, self-mode-locked or hybrid-mode-locked lasers of those types, or soliton lasers.[32] In this case the requirement to the passband of the main maximum appears to be of the opposite character, since shortening of the laser output pulses and further proceeding to the range of subpicosecond and femtosecond pulses is possible only with the appropriate widening of the passband of the intracavity selector. Usually, a single- or multicomponent filter is used designed for the narrowband cw tunable laser, in which only the thinnest plate is retained in the laser cavity to provide maximum widening of the BF passband. However, even with a single plate the linewidth of laser emission appears much narrower than the limiting linewidth which can be achieved with a BF of this type.

Calculations show that the thinnest plate of the conventional BF displays in the 500–700-nm range the main maxima of the fifth–seventh order. Since the spectral passband of the main transmission peak increases with the order decrease [see Eqs. (15) and (18)], the first-order maximum exhibits the largest spectral width. Thus, for a filter produced by Coherent, Inc., (crystalline quartz, thickness 0.35 mm, $\beta=0°$)[32] the calculated bandwidth of the transmission peak of the sixth order at $\lambda=600$ nm is $\simeq 7$ nm, of the fifth order is $\simeq 10$ nm, and of the first order is $\simeq 46$ nm. To obtain the first-order transmission maximum at the indicated wavelength, the thickness of the plate has to be reduced by a factor of 5–7, which is practically impossible. In our opinion, the problem can be solved by choosing the optic axis orientation in the plate (i.e., the angle β) that would provide the first-order selection of the required laser wavelength keeping the plate thickness relatively large. This approach was chosen in Ref. 33.

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